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Enhanced entanglement transfer by phase-shift control in two parallel Heisenberg spin chains

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Abstract

We investigate the effect of phase shift on the entanglement transfer in two parallel 1D spin chains. We calculate the concurrence, measures for two-qubit entanglement, as a function of time. We find the maximum achievable entanglement in the cases with and without phase shift. Although the entanglement transfer becomes slow in most cases with phase shift, the significant enhancement of the maximum achievable entanglement is obtained which suggests its potential usefulness in quantum information processing.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

It is an important task to transfer quantum information from one location to another in quantum information processing. Many methods are proposed for accomplishing this task. For example, it has been investigated by means such as ion trap [1], NMR [2], atoms in a thermal bath of photons [3] and linear optics [4]. Recently quantum spin networks as an ideal communication channel to realize quantum information process have been studied [5–14]. In the first proposal, Bose [5] demonstrated that the quantum spin chain can be used as a channel for short distance quantum communication. A quantum state can be transferred from one end of the chain to the other end. Christandl *et al* suggested a perfect state transfer algorithm which can transfer an arbitrary quantum state between two ends of a spin chain [6] or more complex spin networks [7]. Li *et al* studied the quantum state transfer via a spin ladder as a robust data bus [8]. Their results show that the spin ladder system is a perfect medium through which the interaction between two distant spins can be mapped to an approximate Heisenberg-type coupling. Subrahmanyam *et al* investigated the transport of entanglement through a Heisenberg XY spin Chain [9], and they analytically calculated the two-qubit entanglement

using various measures with different initial states. Also, entanglement transfer in a chain of coupled oscillators [10], measurement-assisted optimal quantum communication by a single chain [11] or parallel spin chains [12, 13], information transfer rates in spin networks [14] have been investigated. Entanglement transfer from continuous variable to finite-dimensional systems [15–17] was also studied. There have been some indications that effective spin networks could be engineered by using arrays of Josephson junctions [18], quantum dots [19], optical lattices [20] or QED cavities [21]. Very recently, the entanglement transfer in a spin chain by phase-shift control [22] was studied. The system consists of a physically isolated (0th) spin and a Heisenberg spin chain with nearest neighbour interaction. Initially the 0th spin and the first spin of the chain are maximally entangled. With the evolution of time, the entanglement of pair of the 0th and the first spins can be transferred to the pair of the 0th and *m*th spins. The maximum entanglement is found to be enhanced significantly by phase-shift control. However, if we want to transfer entanglement from one spin pair to another spin pair for a long distance, we need to use two parallel spin chains. In this paper, we analyse the entanglement transfer in two parallel spin chains by phase-shift control. We show that the maximum achievable entanglement can also be significantly enhanced by phase-shift control through parallel spin chains, but unfortunately in most cases the time we need to obtain it with phase shift is greater than the case without phase shift, indicating the trade-off between the speed and quality of the entanglement transfer with phase shift.

2. The model

The system we consider here consists of two parallel Heisenberg spin chains. Each chain contains N spins. There are no interactions between the two chains. The parallel chains can easily be realized in many experiments [23, 24]. In [23], P Gambardella *et al* have shown that it is possible to produce high-density (5×10^6 cm^{-1}) arrays of one-dimensional (1D) parallel monoatomic chains by growing Co on a high-quality Pt vicinal surface. The 1D geometry of the ferromagnetic chains is manifested by a strong uniaxial anisotropic behaviour. In their experiment, they did not find evidence for inter-chain coupling effects [25] between the chains. In [24], Motoyama *et al* have found that Sr_2CuO_3 can have 1D parallel chains structure. The inter-chain exchange interaction $J_\perp$ is extremely small compared with the intra-chain exchange interaction J, $J_\perp/J \sim 10^{-5}$ [26]. Therefore, this system shows an ideal 1D behaviour of the $S = 1/2$ Heisenberg antiferromagnet. Actually it is easier to produce a whole bunch of parallel uncoupled chains than just a single one in above experiments. The total Hamiltonian of the system can be given by [12, 13]

$$H = H^{(1)} \otimes I^{(2)} + H^{(2)} \otimes I^{(1)}, \tag{1}$$

where $H^{(i)}(i = 1, 2)$ denotes the Hamiltonian of the ith spin-1/2 Heisenberg chain with the nearest neighbour interaction and has the following form:

$$H^{(i)} = -\sum_{j=1}^{N} \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + S_j^z S_{j+1}^z + h S_j^z\right], \tag{2}$$

where $S_j^\alpha(\alpha = x, y, z)$ represents the components of the spin operator and the periodic boundary conditions $\left(S_{N+1}^\alpha = S_1^\alpha\right)$ is satisfied, h is the external magnetic field along the z axis. The coupling constant between the nearest sites is taken as $J = -1$. The two parallel spin chains constitute the physical channel through which the information transfer as shown in figure 1.

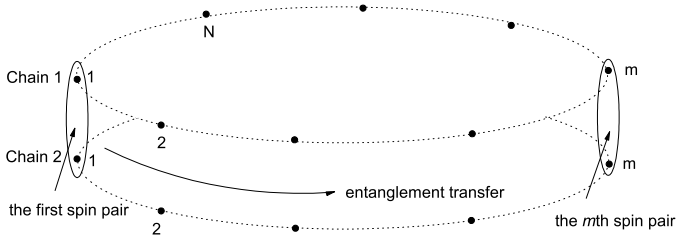


Figure 1. Schematic picture of the system. The two ring-shaped spin chains are parallel with each other and each contains N spins. The periodic boundary condition is satisfied in each chain. There are no interactions between the two chains. Initially, the first spin pair is maximally entangled.

Now suppose that initially the first spin pair is maximally entangled as $1/\sqrt{2}(|0\rangle^{(1)}|1\rangle^{(2)} + |1\rangle^{(1)}|0\rangle^{(2)})$ and all other spins are set in the ferromagnetic ground state $|\mathbf{0}\rangle^{(i)} = |00\ldots0\rangle^{(i)}$. The state

$$|\mathbf{j}\rangle^{(i)} = |0_1 0_2 \ldots 1_j \ldots 0_N\rangle = \sigma_j^+ |\mathbf{0}\rangle^{(i)} \qquad (j = 1, 2, \ldots, N), \tag{3}$$

which denotes the spin at the jth sites has been flipped to the $|1\rangle$ state. The initial state of the whole system will be (for differentiating it with another initial state, we define it as the first initial state)

$$|S(0)\rangle = 1/\sqrt{2}(|\mathbf{0}\rangle^{(1)}|\mathbf{1}\rangle^{(2)} + |\mathbf{1}\rangle^{(1)}|\mathbf{0}\rangle^{(2)}). \tag{4}$$

Here we assume that the total number of up spins in a chain is one which indicates that each chain is in the 'one-magnon' state. Now we consider the influence of phase shift on the each chain's Hamiltonian. It is well known that both the Aharonov–Casher (AC) effect [27] and the Dzyaloshinskit–Moriga interaction [28] can generate phase shift in a spin chain. Though having different producing mechanism, they have same effect on the enhancement of entanglement transfer. Here we consider the AC effect. When a (quasi-)magnetic moment $\overrightarrow{\mu}$ of the spin eigenstates (magnon) travels along a chain in the presence of an electrical field, its wavefunction acquires an extra phase, which is the AC phase

$$\Delta\theta = \frac{1}{\hbar c^2} \int_{\overrightarrow{x}}^{\overrightarrow{x}+\Delta\overrightarrow{x}} \overrightarrow{\mu} \times \overrightarrow{E} \cdot \mathrm{d}\overrightarrow{x}. \tag{5}$$

in addition to the ordinary dynamical phase. It is equivalent to spin–orbit coupling that the particle moving in an electric field feels a magnetic field as well due to the relativistic effect. The applied electric field breaks the rotational symmetry of the system, and the accumulated AC phase along the chain does not vanish after a 2π rotation, thus the dispersion relations is not symmetric with the zero wave number ($k = 0$). The concurrence between any spins can be expected to change accordingly because it is related to the dispersion relations. This is the reason that the AC phase can affect the entanglement transfer [22].

From equation (5), the direction of the external field can be taken to be perpendicular to the direction of the (quasi-)magnetic moment for gaining the largest phase shift θ. For example, if the direction of the magnetic moment is up in the two chains, the electric field can be applied along the radial direction (see figure 2). This configuration is just like an electron in an atom. The electric field could be generated by charges on a wire at the centre of the two rings [22].

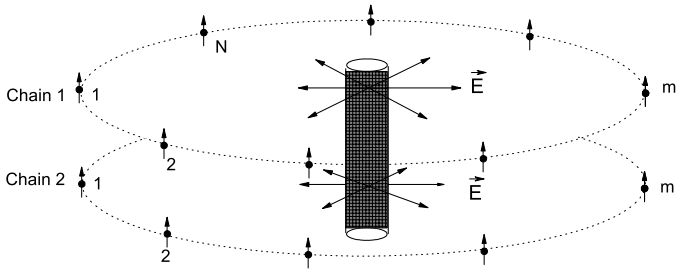


Figure 2. Configuration for the Aharonov–Casher effect in two rings. Assuming the direction of the (quasi-)magnetic moment is up and the electric field is along the radial direction.

When considering the effect of the phase shift, the Hamilton in equation (2) becomes [22, 29, 30]

$$H^{(i)} = -\sum_{j=1}^{N}\left[\frac{1}{2}\left(\mathrm{e}^{\mathrm{i}\theta}S_j^+S_{j+1}^- + \mathrm{e}^{-\mathrm{i}\theta}S_j^-S_{j+1}^+\right) + S_j^zS_{j+1}^z + hS_j^z\right], \tag{6}$$

where θ is the phase change between neighbouring spin which is produced by the AC effect. It can be diagonalized under the Jordan–Wigner transformation [31, 32]. The eigenvalues and eigenstates can be obtained as [22]

$$E_k^{(i)} = -\cos(k+\theta) + \left(1-\frac{N}{4}\right) - h\left(1-\frac{N}{2}\right), \tag{7}$$

$$|\mathbf{k}\rangle^{(i)} = \frac{1}{\sqrt{N}}\sum_j \mathrm{e}^{\mathrm{i}kj}|\mathbf{j}\rangle^{(i)}, \tag{8}$$

where $k = 2\pi n/N$ with $-N/2 < n \leqslant N/2$. We can see that θ only appears in the eigenvalues $E_k^{(i)}$ and it does not affect the eigenstates $|\mathbf{k}\rangle^{(i)}$. Eigenstates $|\mathbf{k}\rangle^{(i)}$ of the Hamiltonian can be looked as the independent quantum state of collective mode. The eigenvalues and eigenstates of the whole system can be expressed as

$$E_k = E_k^{(1)} + E_k^{(2)}, \tag{9}$$

$$|\mathbf{k}\rangle = |\mathbf{k}\rangle^{(1)} \otimes |\mathbf{k}\rangle^{(2)}. \tag{10}$$

With the evolution of time, the excitation in equation (4) will travel along the two spin chains which can be thought as the interference between all modes. Now, a spin up state can be obtained by a spatial Fourier transformation in equation (8),

$$|\mathbf{j}\rangle^{(i)} = \frac{1}{\sqrt{N}}\sum_k \mathrm{e}^{-\mathrm{i}kj}|\mathbf{k}\rangle^{(i)}. \tag{11}$$

Hence, we obtain the state of the system at time t, taking $\hbar = 1$, as

$$|S(t)\rangle = 1/\sqrt{2}\left\{|\mathbf{0}\rangle^{(1)} \otimes \left[\sum_j \lambda_j(t)|\mathbf{j}\rangle\right]^{(2)} + \left[\sum_j \lambda_j(t)|\mathbf{j}\rangle\right]^{(1)} \otimes |\mathbf{0}\rangle^{(2)}\right\}, \tag{12}$$

where

$$[\lambda_j(t)]^{(i)} = \frac{1}{N}\sum_k \exp\left[\mathrm{i}k(j-1) - \mathrm{i}E_k^{(i)}t\right]. \tag{13}$$

We would like to give the measure of entanglement, the concurrence. For a pair of qubits, the concurrence is given by [33] $C_{ij} = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$, where $\lambda_i (i = 1, 2, 3, 4)$ are the square roots of the eigenvalues of the operator $\varrho_{ij} = \rho_{ij}(\sigma_1^y \otimes \sigma_2^y)\rho_{ij}^{\star}(\sigma_1^y \otimes \sigma_2^y)$, with $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \lambda_4$ and ρ_{ij} is the density matrix of the pair qubits; σ_1^y and σ_2^y are the normal Pauli operators. The concurrence $C_{ij} = 0$ corresponds to an unentangled state and $C_{ij} = 1$ corresponds to a maximally entangled state. The concurrence measure entanglement of the Hamiltonian in equation (2) has been investigated [34], and the entanglement dynamics have been discussed [35].

Our task is to calculate how much entanglement exists between the mth spin pair and find the maximum entanglement that can be achieved in a certain time interval. The reduced density matrix of the state of the mth spin pair can be calculated by tracing out the state of all other sites. The concurrence C_m^N (N is the number of sites in each chain and m denotes the mth spin pair) can be obtained as

$$C_m^N = |[\lambda_m(t)]^{(1)}||[\lambda_m(t)]^{(2)}|. \tag{14}$$

It can be seen that the second and third terms in equation (6) are common to all modes, they have no effect to the interference and thus no influence on the entanglement we will calculate. For convenience, we eliminate them later. Because $[\lambda_m(t)]^{(1)}$ and $[\lambda_m(t)]^{(2)}$ have same expressions, we let $\lambda_m(t) = [\lambda_m(t)]^{(1)} = [\lambda_m(t)]^{(2)}$. Then C_m^N can be rewritten as

$$C_m^N = |\lambda_m(t)|^2, \tag{15}$$

where

$$\lambda_m(t) = \frac{1}{N}\sum_k \exp[\mathrm{i}k(m-1) + \mathrm{i}\cos(k+\theta)t]. \tag{16}$$

From the point of view of wave mechanics, we can identify the amplitudes $\lambda_m(t)$ as propagators. Then From equation (15) the concurrence between the mth spin pair equals to square of the absolute value of the propagators. We may expect the constructive interference at the target spin pair by controlling the propagators. From equation (16), we find that the propagators are naturally affected by phase shift θ.

Now let us consider the second initial state of system

$$|S(0)\rangle = 1/\sqrt{2}(|\mathbf{0}\rangle^{(1)}|\mathbf{0}\rangle^{(2)} + |\mathbf{1}\rangle^{(1)}|\mathbf{1}\rangle^{(2)}). \tag{17}$$

Note that there exist two up spins in the whole system (each chain contains one) in this initial state, which is different from the first initial state.

Following the same approach, the concurrence of the mth spin pair can be calculated. We only give the reduced density matrix of the mth spin pair because of the complicated expression of the concurrence, which is given by

$$\rho_m^N = \frac{1}{2}\begin{pmatrix} 1+(1-|\lambda_m(t)|^2)^2 & 0 & 0 & \mathrm{e}^{2\mathrm{i}t}(\lambda_m^*(t))^2 \\ 0 & (1-|\lambda_m(t)|^2)|\lambda_m(t)|^2 & 0 & 0 \\ 0 & 0 & (1-|\lambda_m(t)|^2)|\lambda_m(t)|^2 & 0 \\ \mathrm{e}^{-2\mathrm{i}t}(\lambda_m(t))^2 & 0 & 0 & |\lambda_m(t)|^4 \end{pmatrix}. \tag{18}$$

3. Results and discussion

In figure 3, we plot the concurrence C as a function of time t and phase shift θ. Figures 3(a) and (b) show different features. In figure 3(a), C is symmetric with respect

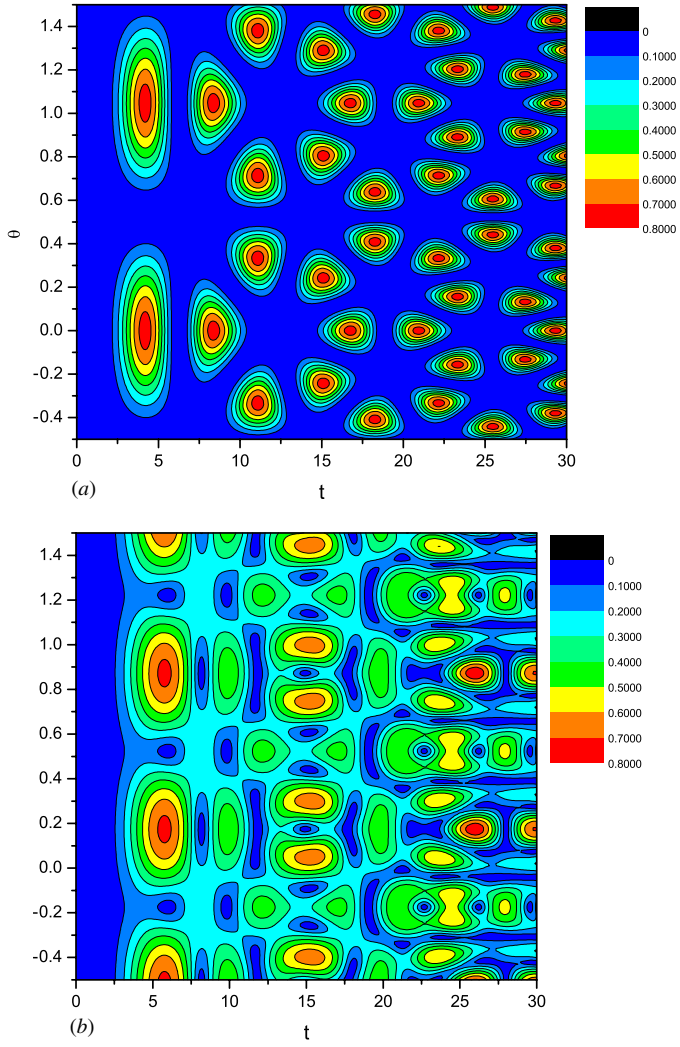
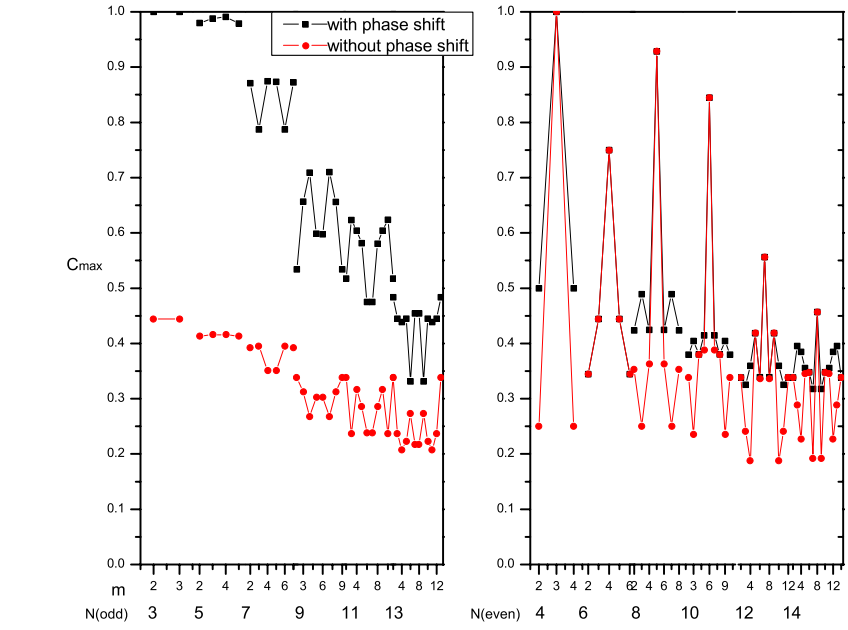


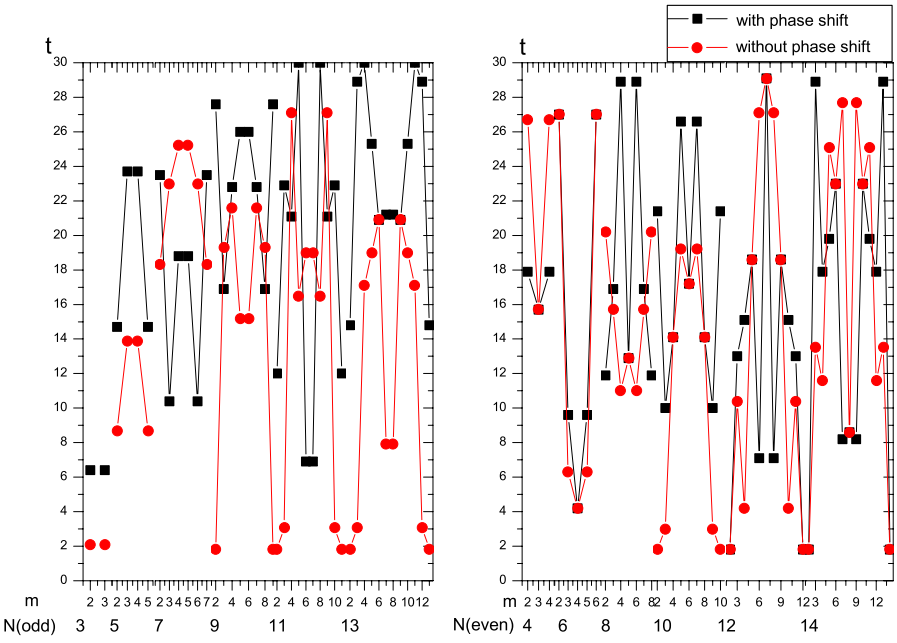
Figure 3. The evolution of the concurrence C with time t in the presence of phase shift. The initial state of the system is $1/\sqrt{2}(|\mathbf{0}\rangle^{(1)}|\mathbf{1}\rangle^{(2)} + |\mathbf{1}\rangle^{(1)}|\mathbf{0}\rangle^{(2)})$. (*a*) $N = 6, m = 4$. (*b*) $N = 9, m = 5$.

to $\theta = 0$ at a certain time t, while in figure 3(*b*) it corresponds to a nonzero θ. Then in figure 3(*a*) the maximum entanglement can be gained at a zero phase shift. This is because an effective constructive interference can even occur at $\theta = 0$ for some (N, m). For $N = 9, m = 5$, figure 3(*b*) clearly shows that the concurrence can be enhanced by a nonzero phase shift θ, when $\theta = 0, C_{\max} = 0.3581$, and when $\theta \neq 0, C_{\max} = 0.5984$. Both are obtained in a time interval $0 \leqslant t \leqslant 30$. We also find that the concurrence demonstrates periodic oscillation behaviour with the phase shift θ at certain time. The concurrence cannot reach to 1 in this condition due to the dispersive free evolution of the chains [5].

Figure 4(*a*) shows the comparison of the maximum entanglement that can be transferred from the first spin pair with and without phase shift. The horizontal axis represents the number of sites N and the corresponding mth spin pair. $C_{\max}$ are found numerically in a time interval $0 \leqslant t \leqslant 30$ and the phase shift $\theta \in [-\pi, \pi]$. The sites $N = 3, 4, \ldots, 14$ and for each



(*a*)



(*b*)

Figure 4. (*a*) The maximum entanglement that can be obtained with phase shift (square plots) and without phase shift (circle plots). The horizontal axis represents the total number of sites N in each chain and mth spin pair that the concurrence is evaluated. (*b*) The time at which the concurrence achieves its maximum value in this system. We note that the time with phase shift is greater than the time without phase shift in most cases, indicating the aforementioned trade-off between the speed and the quantity of the transferred entanglement. The initial state of the system is taken as in figure 3. The results are obtained in a time interval $0 \leqslant t \leqslant 30$ and the phase shift $\theta \in [-\pi, \pi]$.

$N, m = 2, 3, \ldots, N$. Figure 4(a) reveals various interesting features. First of all, we find that the maximum entanglement with phase shift is greater than or equal to the one which is obtained without phase shift. C_2^3, C_3^3, C_3^4 can reach to 1 with phase shift, i.e., the perfect entanglement transfer can be realized. Second, the maximum entanglement only depends on the relative location of the two spin pairs, i.e., the two maximum values which are obtained at the mth and $(N + 2 - m)$th spin pairs are equal. It can be seen clearly from equation (16) that replacing the term $(m - 1)$ with $N - (m - 1)$ does not change the result. Finally, the enhancement of entanglement transfer with phase shift is more evident for odd chains. That is to say, an odd N(an odd modes) in the chains can give a more effective interference. For a fixed N, there exist two peaks when N is odd. The location of the two peaks is not certain. For example, they are at $m = 4, 7$ when $N = 9$, while $m = 3, 10$ when $N = 11$. However, when N is even, only one peak exists which locates at $m = N/2 + 1$ and it occurs at a zero phase shift. With the increasing of N, the maximum entanglement decreases except for $N = 6$ in which a less constructive interference occurs. Unfortunately, we find that in most cases transfer seems to be much slower when gaining the enhancement of entanglement with phase shift. In figure 4(b), the time at which the concurrence achieves its maximum value in the interval $0 \leqslant t \leqslant 30$ is plotted. We find that the time with phase shift is greater than the time without phase shift in most cases. So there appears to be some trade-off between the speed and quality of the entanglement transfer. Suppose the maximum entanglement with and without phase shift is obtained at (θ_1, t_1) and $(\theta_2 = 0, t_2)$, respectively; it is possible to gain an effective constructive interference at a shorter time $(t_1 < t_2)$ with a proper nonzero phase shift θ_1 at some (N, m). For example, when $N = 4$, $m = 2, 4$ or $N = 7$, $m = 3, 4, 5, 6$, the better compromise between the quality and speed of the entanglement transfer with phase shift is obtained. Here we only consider the inter-chain entanglement transfer. Can the entanglement be transferred from the first spin pair to the arbitrary two spins which belong to the same chain? We have explored this question in the simplest case (each chain contains two spins) and found that the intra-chain entanglement only depends on the intra-chain coupling [36]. The intra-chain entanglement has no certain relations with the initial inter-chain entanglement. We believe that it is also true when each chain contains N spins.

Figure 5 shows the comparison of the maximum entanglement $C_{\max}$ between the first and the second initial states. We can see that $C_{\max}$ gained using the second initial state is always lower than using the first initial state, though they show the same oscillatory behaviour with increasing m. Then we can conclude that using the first initial state is more suitable for quantum entanglement transfer in this system.

In the thermal dynamical limit $(N \to \infty)$, the wave number k becomes continuous. We can replace the summation in $\lambda_j(t)$ with integration through the relations

$$\frac{1}{N}\sum_k \to \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{d}k. \tag{19}$$

Then the entanglement of arbitrarily mth spin pair can be calculated numerically. In this limit, the effect of the phase shift disappears. This can be explained that the energy spectrum becomes continuous with large N and the effect of phase shift is same to every mode. The total effect of phase shift only gives a global phase factor and it does not affect the entanglement transfer [22]. In figure 6, we plot the maximum entanglement as a function of the mth spin pair in the thermal dynamical limit. For two different initial states, the maximum entanglement both exponentially decay with increasing m. The oscillatory behaviour of the maximum entanglement with increasing m, when N is small, disappears. For the second initial state, the entanglement decays to zero quickly. Thus, the entanglement cannot be transferred to a long distance in this condition. However,

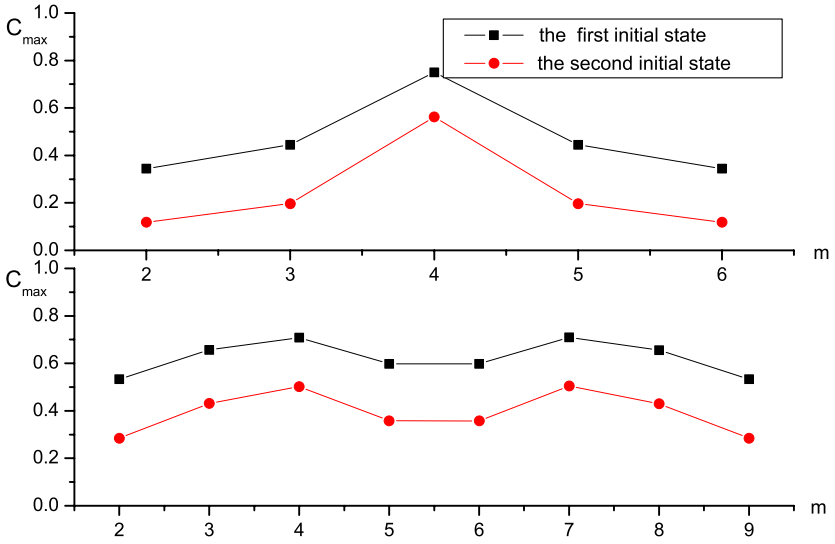


Figure 5. Comparison of the maximum entanglement with phase shift between the two initial state for $N = 6, 9$. The range of the time and phase shift is same with figure 4.

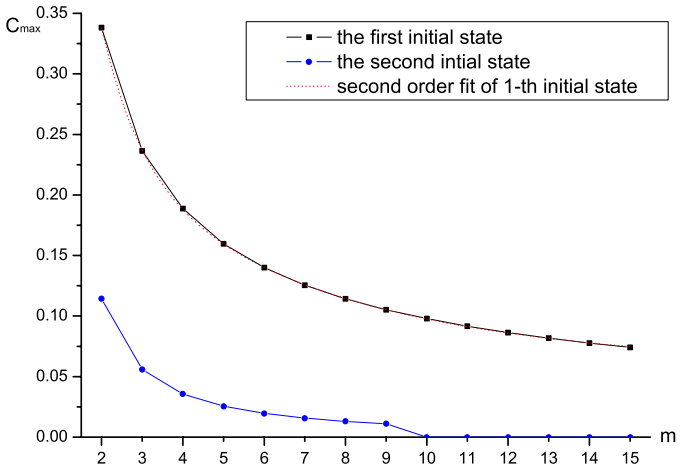


Figure 6. The maximum entanglement as a function of the mth spin pair for different initial states in the thermal dynamical limit($N \to \infty$). The dotted line is the second-order exponential fit to points on the first initial state.

for the first initial state, we make a second-order exponential fit to the points and obtain a relation $C_{\max} = 0.90\exp[-m/0.97] + 0.24\exp[-m/5.46] + 0.06$. Thus in the limit $m \to \infty$, $C_{\max} = 0.06$. The entanglement can be transferred to the last spin pair, though it is very small.

4. Conclusion

We have investigated the effect of phase shift on the entanglement transfer via two parallel spin chains. We find that, in general, the maximum entanglement obtained with phase shift is

greater than or equal to the one which is obtained without phase shift. For a fixed small N, when N is odd, the maximum entanglement can be enhanced more evidently by phase-shift control, while when N is even, the position of the maximum entanglement locates at $m = N/2 + 1$ and it is not affected by the phase-shift control. Unfortunately, it will take more time to obtain an enhanced maximum entanglement in most cases. In the thermal dynamical limit, the effect of phase shift disappears and the maximum entanglement exponentially decays with increasing m. However, the gap between the theory and experiment may be difficult to bridge. As pointed in [22], there could be a difficulty in providing an electric field that is intense enough to generalize a meaningful phase shift. A rough estimation of the necessary strength of the electric field is at least 10^7 V m^{-1} and such a strong electric field can be realized by two-dimensional electric gases formed in heterostructured SiGe, GaAs, or other types of III–V materials. With the development of nano-structure fabrication techniques, we expect the phase shift will be useful in the quantum information processing.

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